



education equals

**Topic: Mixed Applications of Networks**

Time: 45 mins

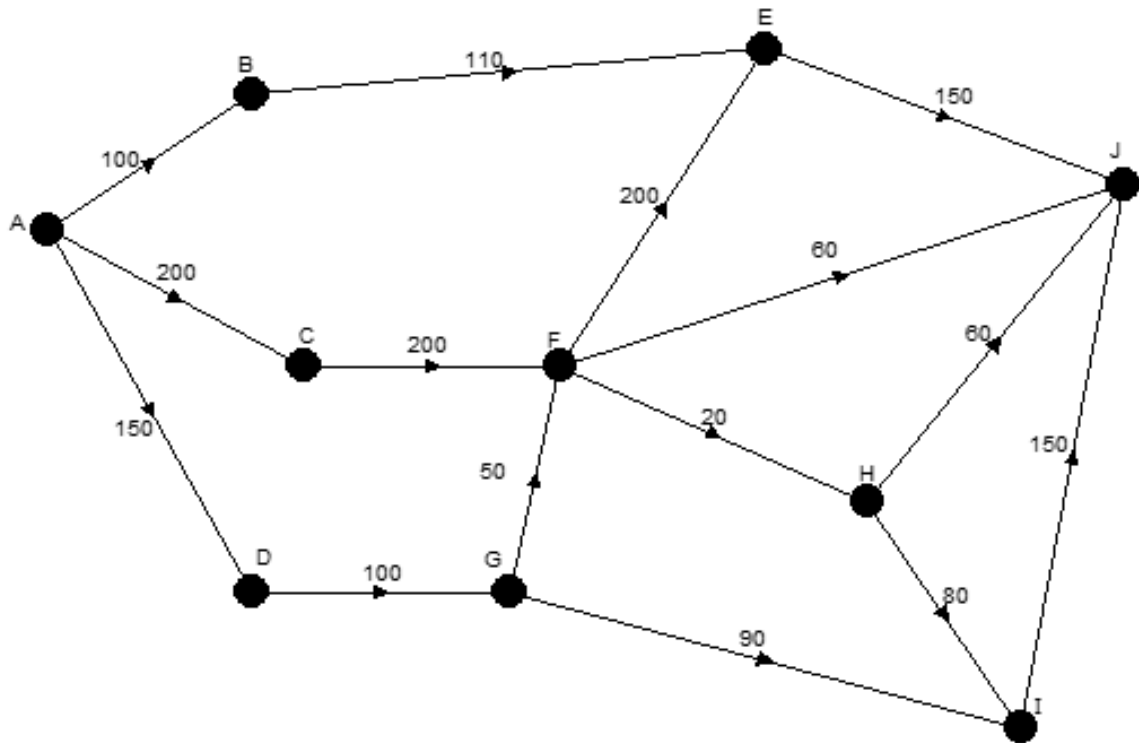
Marks:

/45 marks

**No calculator allowed**

**Question One: [4, 2: 6 marks]**

a) Find the maximum flow of the network below.



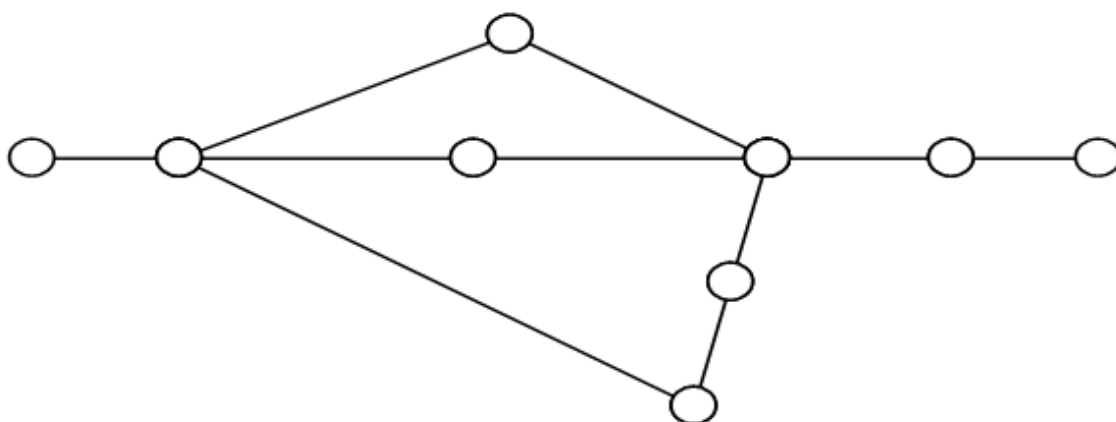
b) Show that the maximum flow is equal to the minimum cut.

**Question Two: [4, 2, 2, 3, 2: 13 marks]**

Borat is planning his next holiday. Once he has decided that he wants to go on holiday the following things need to happen before he can go.

Activity	Preceding activity	Time to complete activity
Pick a destination	None	2 weeks
Get relevant vaccinations and health checks	Pick a destination	3 days
Apply for passport	Pick a destination	6 weeks
Start saving money	Pick a destination	
Apply for a visa	Apply for a passport	2 weeks
Travel insurance	Get relevant vaccinations and health checks	3 days
Buy a plane ticket	Start saving	1 day
Change currency	Buy a plane ticket	1 day
Pack suitcase	Travel insurance, Apply for visa, Change currency	2 days
Go to the airport and get on plane	Pack suitcase	1 day

- a) Add the information from the table to the project network below, complete the project network and calculate the number of days Borat allocated to saving money if it takes 100 days before he leaves for his holiday.



[Note: The diagram below is not drawn to scale]

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- b) What is the latest possible day he can start getting his health checks and vaccinations without delaying leaving for his holiday?

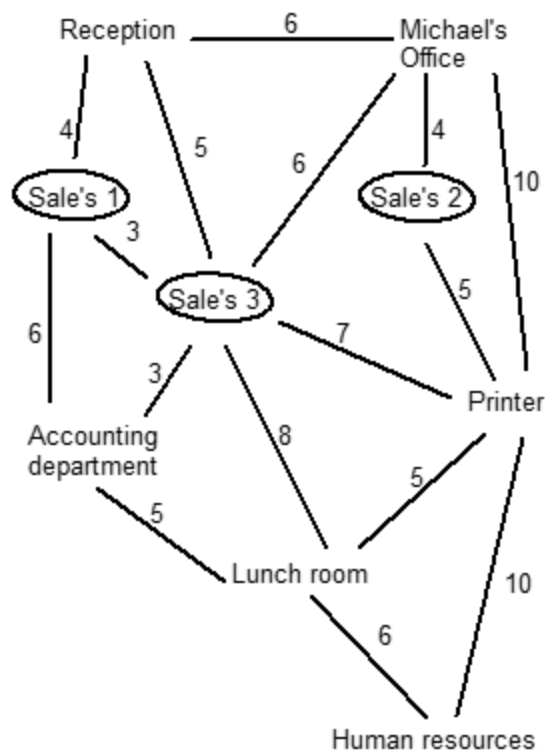
After deciding where is going to travel to Borat begins collecting donations to take to an orphanage in a village he will visit.

- c) How long can he collect donations for without it delaying his departure?
- d) If Borat finishes all his relevant health checks on July 14<sup>th</sup> and uses all the float time allowed for this activity, what date does he go to the airport to get on the plane?
- e) If there is a delay in the processing of his visa application and it takes an extra four weeks to process, how does this affect how long it takes before he can leave for his holiday?

**Question Three: [3, 3, 3: 9 marks]**

Michael is having trouble with the wifi in his office and so he has decided to hardwire all the work stations.

The office consists of a reception desk, the manager Michael's office, three separate sale's teams, an accounting department, printer station, lunch room and human resources. The follow network depicts the distance in metres between the work stations.



- a) Find the shortest path from the reception to human resources. State the path and the distance.

To hardwire all the workstations, the workstations need to all be connected by cabling.

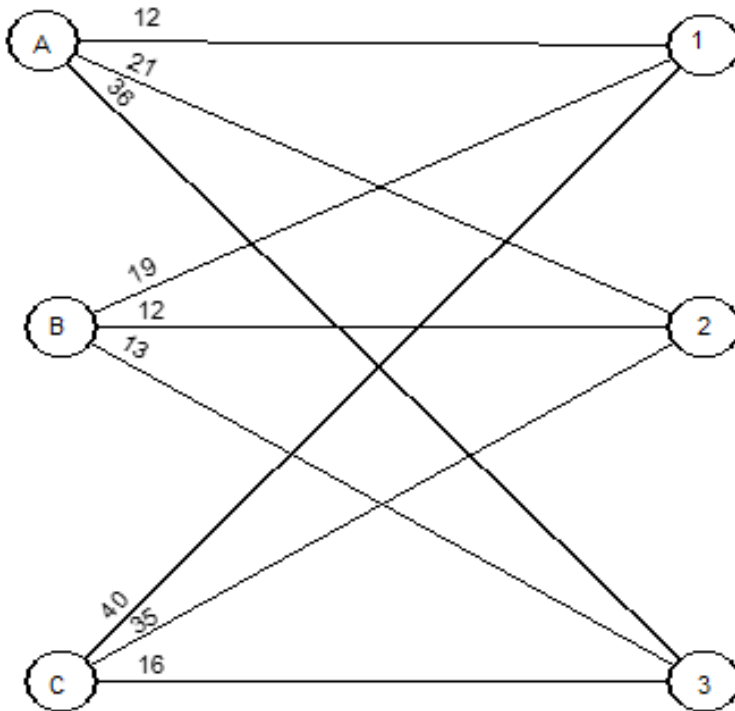
- b) Show span of the cabling on the diagram above and state the minimum amount of cabling needed to complete the job.

The cost of the cabling including labour is approximately \$50/metre.

- c) How much money do they save by not cabling the lunch room.

**Question Four: [2, 2, 2, 1: 7 marks]**

Consider the following complete bipartite graph.

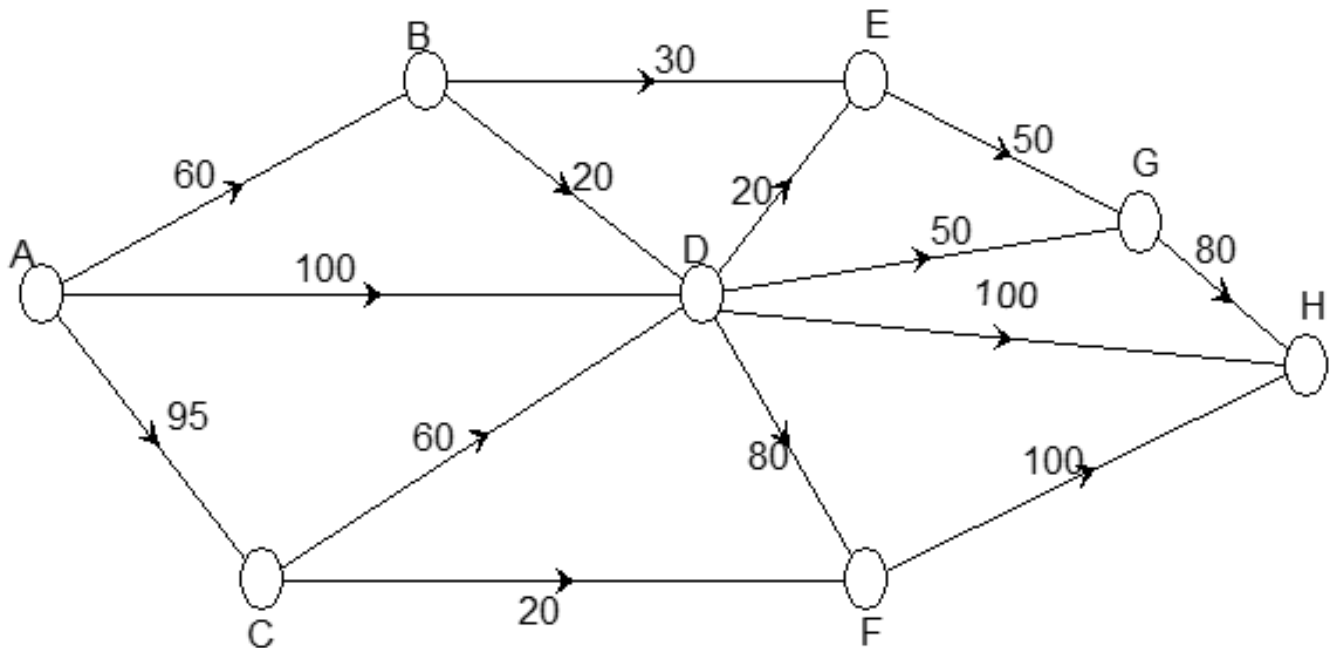


- Create a matrix to show the weighting of each allocation.
- Assign tasks A, B, C to 1, 2, 3 in order to maximize the weightings.
- Assign tasks A, B, C to 1, 2, 3 in order to minimize the weightings.
- What is the difference between the maximum and minimum weightings?

**Question Five: [1, 4, 2, 3: 10 marks]**

The network below represents a system of roads through a major city district. The council has tried to limit the traffic flow through this part of the city and therefore there is only one entry and a single exit point. The arcs on the network represent the roads and the weightings on the arcs give the maximum capacity of cars for that section of road in cars per hour.

- a) Label the source and the sink.



- b) What is the maximum number of cars that could reach point H per hour given that the only restriction is the capacity of the roads?

- b) Show that the maximum flow is equal to the minimum cut.

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One of the three roads which currently has a capacity of 20 cars per hour will be upgraded to be able to accommodate a maximum of 60 cars per hour.

- c) Describe the effect on the maximum flow for each of the roads in question and decide which one they should upgrade, if any.



**Topic: SOLUTIONS**

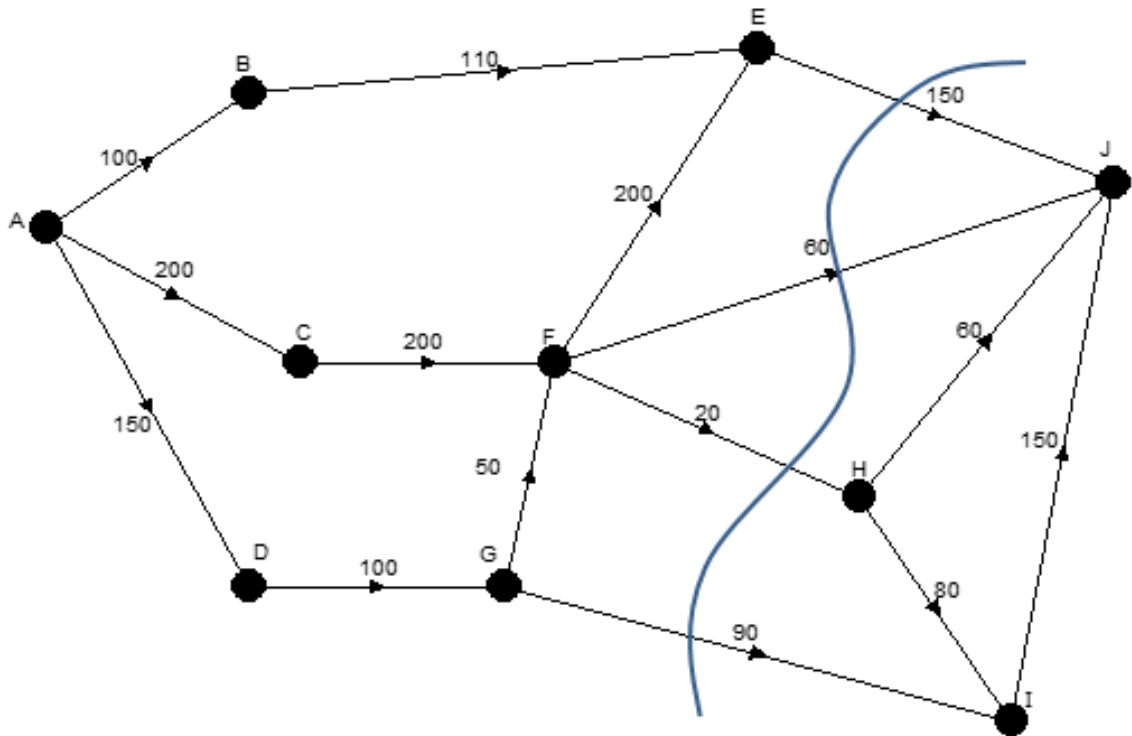
Time: 45 mins

Marks: /45 marks

**No calculator allowed**

**Question One: [4, 2: 6 marks]**

a) Find the maximum flow of the network below.



*ABEJ* 100

*ACFEJ* 50 ✓

*ACFJ* 60 ✓

*ACFHJ* 20 ✓

*ADGIJ* 90

*Total: 320* ✓

b) Show that the maximum flow is equal to the minimum cut.

*E - J* 150      *F - H* 20

*F - J* 60 ✓      *G - I* 90      *Total 320 = maximum flow* ✓

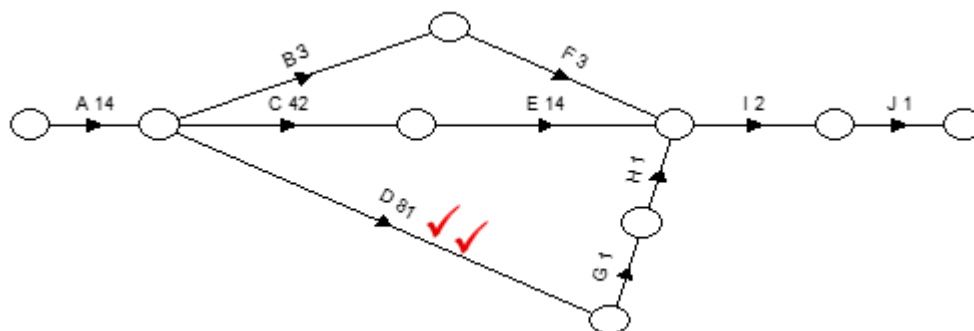


**Question Two: [4, 2, 2, 3, 2: 13 marks]**

Borat is planning his next holiday. Once he has decided that he wants to go on holiday the following things need to happen before he can go.

	Activity	Preceding activity	Time to complete activity
A	Pick a destination	None	2 weeks 14 days
B	Get relevant vaccinations and health checks	Pick a destination	3 days
C	Apply for passport	Pick a destination	6 weeks 42 days
D	Start saving money	Pick a destination	81 days
E	Apply for a visa	Apply for a passport	2 weeks 14 days
F	Travel insurance	Get relevant vaccinations and health checks	3 days
G	Buy a plane ticket	Start saving	1 day
H	Change currency	Buy a plane ticket	1 day
I	Pack suitcase	Travel insurance, Apply for visa, Change money	2 days
J	Go to the airport and get on plane	Pack suitcase	1 day

- a) Add the information from the table to the project network below, complete the project network and calculate the number of days Borat allocated to saving money if it takes 100 days before he leaves for his holiday.



[Note: The diagram below is not drawn to scale] arrows ✓ tables and time ✓

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- b) What is the latest possible day he can start getting his health checks and vaccinations without delaying leaving for his holiday?

The latest day is 91 days after starting. ✓ ✓

After deciding where is going to travel to Borat begins collecting donations to take to an orphanage in a village he will visit.

- c) How long can he collect donations for without it delaying his departure?

For 86 days ✓ ✓

- d) If Borat finishes all his relevant health checks on July 14<sup>th</sup> and uses all the float time allowed for this activity, what date does he go to the airport to get on the plane?

✓ 94 days after July 14<sup>th</sup> – July has 31 days total so 17 days left. Plus August 31 days. Plus September has 30 days. → 78 days ✓

$78 + 16 = 94 \therefore$  16<sup>th</sup> October he goes to the airport. ✓

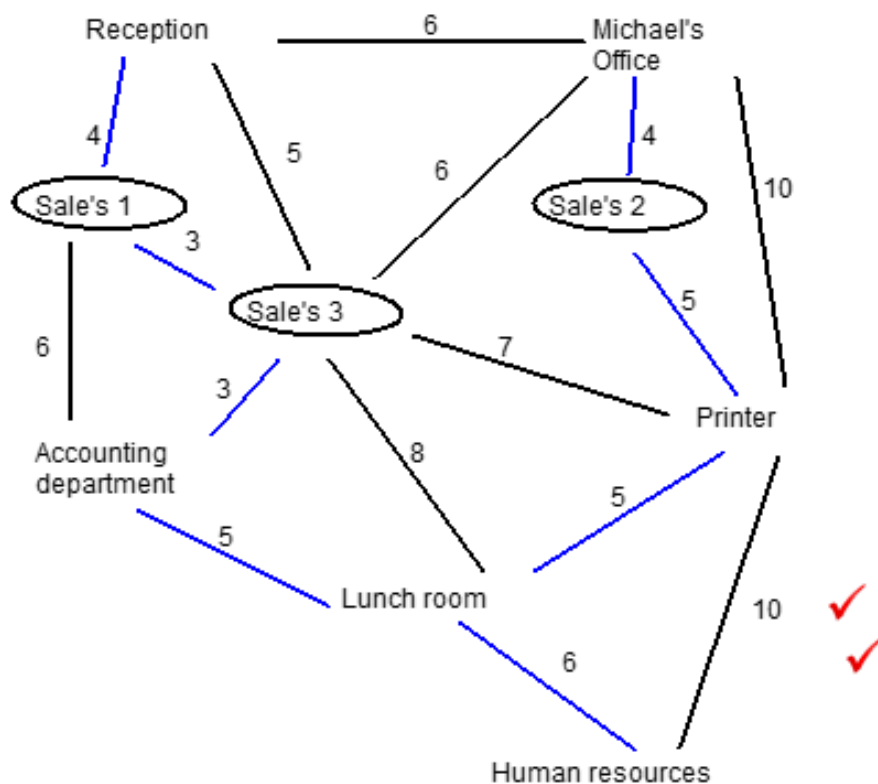
- e) If there is a delay in the processing of his visa application and it takes an extra four weeks to process, how does this affect how long it takes before he can leave for his holiday?

It adds 1 days before he can leave. In reality this delay would cause him to miss his original flight. ✓ ✓

**Question Three: [3, 3, 3: 9 marks]**

Michael is having trouble with the wifi in his office and so he has decided to hardwire all the work stations.

The office consists of a reception desk, the manager Michael's office, three separate sale's teams, an accounting department, printer station, lunch room and human resources. The follow network depicts the distance in metres between the work stations.



- a) Find the shortest path from the reception to human resources. State the path and the distance.

$$R - S3 - LR - HR = 19 m \quad \checkmark \checkmark \checkmark$$

To hardwire all the workstations, the workstations need to all be connected by cabling.

- b) Show span of the cabling on the diagram above and state the minimum amount of cabling needed to complete the job.

$$35 m \quad \checkmark$$

The cost of the cabling including labour is approximately \$50/metre.

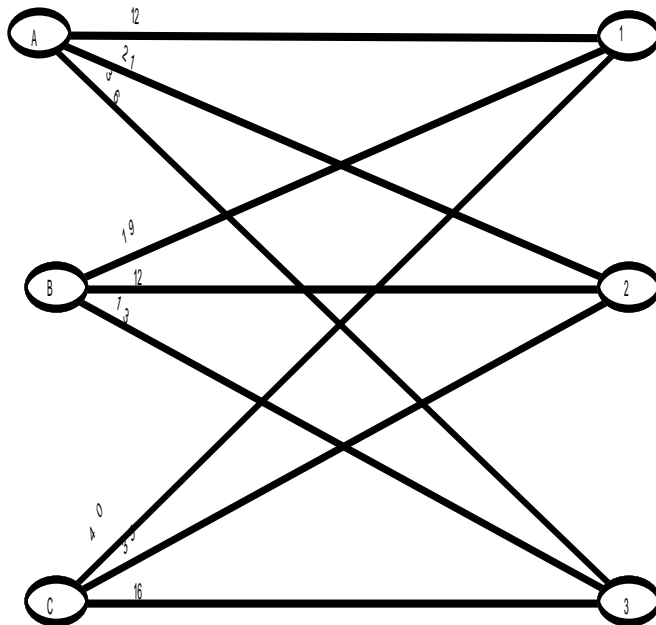
- c) How much money do they save by not cabling the lunch room. *If they don't cable the lunch room human resources still needs to be connected by cabling.*  $\checkmark$

$$\text{Remove Acc} \rightarrow LR, LR \rightarrow HR \text{ and Pr} \rightarrow LR: -5, -6, -5 = -16 \quad \checkmark$$

$$\text{but add Pr} \rightarrow HR + 10 \text{ add Recp} \rightarrow \text{Michael} + 6 \therefore \text{overall zero change} \quad \checkmark$$

**Question Four: [2, 2, 2, 1: 7 marks]**

Consider the following complete bipartite graph.



a) Create a matrix to show the weighting of each allocation.

	<i>A</i>	<i>B</i>	<i>C</i>	
1	12	21	36	✓
2	19	12	13	✓
3	40	35	16	

b) Assign tasks A, B, C to 1, 2, 3 in order to maximize the weightings.

*A* – 3    36

*B* – 1    19    ✓

*C* – 2    35

*Total*: 90    ✓

c) Assign tasks A, B, C to 1, 2, 3 in order to minimize the weightings.

*A* – 1    12

*B* – 2    12    ✓

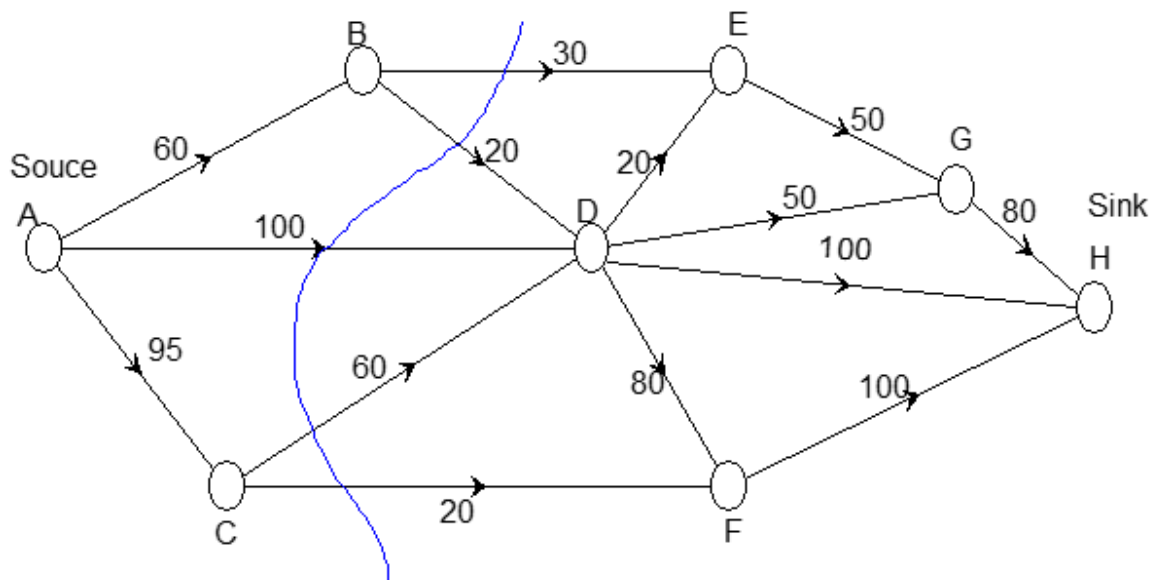
*C* – 3    16

*Total*: 40    ✓

d) What is the difference between the maximum and minimum weightings? 50 ✓

**Question Five: [1, 4, 2, 3: 10 marks]**

The network below represents a system of roads through a major city district. The council has tried to limit the traffic flow through this part of the city and therefore there is only one entry and a single exit point. The arcs on the network represent the roads and the weightings on the arcs give the maximum capacity of cars for that section of road in cars per hour.



a) Label the source and the sink.



b) What is the maximum number of cars that could reach point H per hour given that the only restriction is the capacity of the roads?

*ABEGH – 30*

*ACDH – 30*

*ABDEGH – 20*

*ACDFH – 30*



*ADGH – 30*

*ACFH – 20*



*ADH – 70*



*Maximum Flow: 230 cars per hour*



c) Show that the maximum flow is equal to the minimum cut.

$$BE + BD + AD + CD + CF = 30 + 20 + 100 + 60 + 20 = 230$$



One of the three roads which currently has a capacity of 20 cars per hour will be upgraded to be able to accommodate a maximum of 60 cars per hour.

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- d) Describe the effect on the maximum flow for each the roads in question and decide which one they should upgrade.

*Upgrading BD → increases max flow by 40*

*Upgrading DE → no effect*



*Upgrading CF → increases max flow by 10*

*∴ Could upgrade BD or CF*

